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ABSTRACT

With respect to the innovative roles of technology within the educational realm, an important task of educational research is the investigation of how school children accommodate themselves to innovative computer-based learning environments. This paper describes the strategies and techniques employed and extended by above-average second- through fourth-grade students in a suburban Tel Aviv school within a self-paced learning environment of new arithmetic concepts and algorithms under the guidance of a widely used computer-managed learning system, the Computer-Based Practice in Arithmetic. These high-achieving students managed to provide correct responses to exercises that they did not fully understand, nor to which they had previously been exposed, by the application of resourceful problem-solving methods. Descriptions and examples are provided for the following observed student strategies: (1) analogy by recognition of a pattern within structure or notation; (2) analogy by generalization of notation, operation, or rule to novel situations; (3) synthesis of previously relevant knowledge or rules; (4) reduction of the observed difference between problem statement and problem goal through the use of means-ends analysis; (5) examination, and subsequent utilization, of special cases or examples; (6) use of heuristics to eliminate computational procedures deemed inappropriate in attempting to shorten the search for correct solutions; (7) use of probabilistic considerations when faced with a reduced list of options; (8) induction from trial-and-error procedures; and (9) acquisition of assistance outside of the computer-based learning environment. The fact that students actively learn and attain satisfactory conceptual knowledge within this system is noteworthy because the system is primarily designed to provide drill and practice as a complement to class instruction rather than being the supplement of innovative and substantive educational materials. (JJK)

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STRATEGIES FOR SELF-REGULATED LEARNING FROM COMPUTERIZED PRACTICE IN ARITHMETIC

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Strategies for Self-Regulated Learning from Computerized Practice in Arithmetic

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In a rapidly changing world, a most important task of educational research is to investigate how children adjust themselves to novel learning environments. This paper describes how good students self-regulate their learning of new arithmetic concepts and algorithms in a learning environment induced by a widely used computer-managed system, named here CBPA (Computer-Based Practice in Arithmetic). The fact that students actively learn in that system is noteworthy because the system is designed to provide drill as a complement to class instruction rather than to teach new material.

To satisfy the primary objectives of individualized learning, the CBPA system enables each student to advance in the levels of practice at his or her own rate. Because each student advances at different rates in several different strands (arithmetic topics), a large proportion of the above-average achieving students receive practice in advance of the concurrent class material. Thus, many students, mostly the better learners, receive CAI exercises whose solution requires knowledge of concepts and algorithms not yet available from class instruction. Observations in a previous study (Hativa, 1988a) reveal that high-achieving

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students use a wide range of strategies for dealing with the problems encountered in the CAI work. This study has been designed to further identify and to categorize these strategies.

Method

For investigating students' solution processes, this study used the naturalistic method of observations coupled with individual interviews. Subjects were the 20% highest-achieving students (as measured by the respective computer class reports) in each of six classes. The classes comprised of 2nd through 4th grades in a Tel Aviv suburban school for medium to high SES population. Each of the subjects was observed by one of two observers during at least five CAI ten-minute sessions, once in every 6-8 consecutive sessions, during a four-month period. The observer recorded on paper every exercise that the student received, all the steps of the student's solution and the computer response. Immediately after the computer session, the observer interviewed the student and asked him or her to describe and explain the method for solution for each type of exercise and the source of the student's knowledge of how to solve this type of an exercise.

It is human nature that a person tries to make the best of a situation. Our observations and interviews reveal that "the name of the game" for high achieving students is the fastest advancement in the hierarchical levels of the CAI

system, in spite of the fact that "success" on a problem does not necessarily represent mathematical understanding. However, conclusions that children fail to develop mathematical understanding because of the teaching environment are premature. This is powerfully illustrated by the fact that many high achievers manage to use the particular CAI environment in ways that first lead to their technically identifying algorithms for solution without understanding and eventually lead to their remarkable learning with understanding of mathematical algorithms and concepts. These points are illustrated and discussed below.

Strategies That Motivated Students Use for "Technical" Advancement in the Hierarchical Levels of Practice

High achieving students manage to provide correct answers to exercises that they do not fully understand. This is achieved by applying strategies that are, in fact, resourceful problem solving methods.

Of the general (as opposed to domain specific) problem-solving strategies identified in research literature, the ones relevant to this work are: problem decomposition; means-end analysis; planning; analogy; checking the solution; comparing solutions with worked examples (Gick, 1986); and embedding the problem into a larger one (Clement, 1984).

Our observed students used many of these listed strategies and a few additional ones, which are described below.

1. Using analogy

"Learning by analogy is the mapping of knowledge from one domain over to the target domain, where it is applied to solve problems." (VanLehn, 1986, p. 152). In the CBPA system, students use frequently solving-by-analogy strategies, particularly in the domain of decimals and negative numbers. Students accomplish this by searching in memory for similar operations, algorithms, patterns, or rules with whole numbers; performing the required operations with whole numbers; and then adjusting the answers to the particular domains by adding the decimal point or the negative sign respectively.

1.1 Analogy by keeping a pattern

(a) keeping the pattern of arithmetic operations to new notations

Example: Most students solved correctly horizontal exercises such as: "0.2+0.3=?" and "(-1)+(-4)=?", or exercises such as "0.04+0.39=?" and "0.921-0.589=?" that were written vertically, the first time that the students received them. This was achieved by analogy to addition and subtraction with whole numbers.

(b) keeping the pattern of a sequence to new notation

Example: all of the high achievers observed solved correctly exercises of the type: -45, -40, ? , -30, -25 or 1.5, 1.6, 1.7, ?, 1.9. They explained that they followed the pattern with whole numbers and then added the "minus sign" or the "period" in order "to make the added number look like all other numbers" in that sequence.

1.2 Analogy by generalization to new situations

(a) generalizing notations to new situations

Example: A fourth-grade student learned from final computer answers to solve correctly exercises of the type:

$$\frac{53}{100} = 0.53$$

A week later he received a new type of exercise consisting of a two-digit number to be divided by 1000. His first answer was:

$$\frac{53}{1000} = 00.53$$

His explanation was that he compared this problem to the previous type that included two zeros in the "100" and one zero in the answer. Thus, he assumed that each additional zero in the denominator required an additional zero before the "period". This explanation reveals the strategy of search for a similar problem and the generalization of the decimal notation to the new situation. His erroneous answer reveals his lack of understanding of the decimal point concept.

(b) generalizing operations to new situations

When students receive new types of exercises that they do not know how to solve but that involve the use of arithmetic operations, students activate knowledge of arithmetic operations with whole numbers.

Example: $\frac{2}{3} + \frac{5}{6} = \frac{?}{?}$ Many students tried: $\frac{7}{9}$ $\frac{2+5}{3+6}$ that is, the students added numerators and denominators separately.

(c) generalizing rules to new situations

Example: A fourth grader received a new type of exercise which required rounding 3.5 to a whole number. She immediately typed a "4". Later she explained that she had already practiced rounding of 35 to tens (ans. 40) and 350 to hundreds (ans. 400) and in the case of 3.5 she applied the same rule.

2. Synthesizing previously known rules

When students receive a new, unfamiliar type of an exercise, they activate a search for relevant knowledge in memory. In using this process, students may identify several different rules that the students then synthesize, in order to solve the new problem.

Example: A fourth grader who, through the CAI system, practiced negative numbers without fully understanding this concept, received a new type of exercise which required completing the missing number in " 4 + ? =0 ". She

immediately typed as an answer a "(-4)". She explained that she used her experience with a previous sequence of exercises which required the adding of a positive whole number to a negative whole number (e.g., "5+(-3)=?"). She remembered that in that sequence, whenever she added two numbers which were "positive and negative of the same number" (e.g., "3+(-3)=?"), the answer was "0". Thus, for a successful solution, she synthesized her knowledge from that previous experience with her knowledge of the logical rule for solving equations that she had already practiced with positive numbers: "given that a+b=c and a+?=c then b is the missing part".

3. Using means-end analysis

Means-end analysis is the strategy of reducing the difference between the current state and the goal of the problem by applying appropriate problem-solving operators (Gick, 1986). In order to use this strategy, students should be provided with the answer or the goal of the problem. Only a small proportion of exercises in the CBPA curriculum provide a final answer or an explicit goal. This occurs primarily in exercises that provide multiple-choice options for answers.

Example: A bright second grader received the following type of exercise which required choosing the right option for an answer:

If 40 + a = 63 then:
1. a = 63 + 40
2. a = 63 - 40
3. a = 40 - 63

These exercises were designed to practice the rules of equality--that is, that one can subtract the same number (40) from both sides of an equation and get an equivalent expression. The observed student who had not studied these rules explained his correct solution as follows: "The computer asks me what number I should add to 40 in order to get 63. This is very easy--I know that I have to add 23. Then I compute the result of each of the three given answers and see which of these gives me 23. 63+40 is too large but 63-40 gives me the right answer". This student compared the given and the expected results and worked to discover a method to reduce the differences between them.

4. Examining special cases

This strategy is primarily used in exercises designed to examine knowledge of rules of arithmetic (e.g., operations with units of computations--"0" and "1"; and the commutative, associative, and distributive laws). A typical question would be: "Is the equality: aXb:c = a:bXc true for every a,b,c?" or: "If a-z=a then (choose the correct answer) 1. z=0; 2. z=1; 3. z=a". Many of the observed students who had not yet studied these rules in their class arithmetic, substituted the English letters in the equation with numbers (e.g., in the first example, a with 1, b with 2 and c with 3); computed mentally (1X2:3=2/3 and 1:2X3=3/2); and made conclusions on the basis of the results of computations.

5. Embedding the problem into a larger one

A third grader received, for the first time, exercises of the type: "identify which one of four two-digit numbers is divisible by 4". The first two groups of exercises were: "62, 81, 72, 69" and "59, 70, 78, 96". For both groups he immediately typed the correct solutions (72 and 96 respectively). He explained later that these numbers were multiplications of 8 and therefore they should also be multiplications of 4. This student embedded the problem of division by 4, a multiplication table that he remembered by heart up to 40, into the problem of division by 8, a multiplication table that he remembered by heart up to 80. It was easy for him to compute mentally very quickly that 96 is also a multiplication of 8 (80+8+8).

6. Using heuristics to shortcut the search for solutions

A few good students, when faced with an impasse, used heuristics to eliminate computational steps that seemed inappropriate, in order to shortcut the search for the correct solution. These heuristics were based either solely on knowledge of arithmetic operations or on familiarity with the way the particular CAI curriculum worked. The students knew what level of difficulty of mental computations they could expect from the CAI curriculum, considering the fact that making computations on paper while on-line was forbidden. A typical rationale provided by students using

heuristics was "it looks logical to try this operation" or "the computer cannot ask me to do such a difficult computation".

Example: A fourth grader, after practicing a sequence of exercises with multiplication of exponents ("$5^3 \times 5^4 = 5^7$"), received for the first time division of exponents: " $5^7 : 5^4 = 5^?$ ". She typed correctly a "3" on the first trial (the exponent, 3, is computed by: 7-4). She explained that she did not know why this answer was correct but she assumed that because addition (3+4) worked well in the multiplication exercises, it could not also be good for division. Next she considered the division of 7 by 4 but rejected this option because "this division was not good" (that is, it did not yield a whole number that she believed was expected by the computer as an answer). Thus, after the mental elimination of addition and division she started with subtraction.

7. Using probabilistic considerations

In contrast with medium-to-low achievers who were observed working with the same CAI system, the observed high achievers only seldom tried to guess when they knew neither the answer nor the algorithm for solution. However, when the better students were faced with the need to choose the correct answer out of a few given options and they did not know the underlying method, they often tried educated guesses.

Example: A third grader received exercises that required identifying out of four three-digit numbers the one divisible by 9 (e.g., "277, 530, 450, 624)". This pupil did not know the rules of divisibility by 9. He did not even know that such rules existed. Thus, he tried to divide mentally each of the four given numbers by 9. Because of the time limitation, he missed frequently the correct answer. Thus, he developed an heuristic procedure for reducing the amount of numbers to be divided by 9. This procedure enabled him to arrive frequently at the correct answer within the short time framework. Step 1 was--looking for a number that ends with a "0" because, according to his explanation, "these numbers are easy to divide". Thus, in the example above, he first tried 530:9, immediately knew that 53 was not divisible by 9, and then he tried 450:9 which worked. In step 2 he used a probabalistic strategy, as follows: if there were two out of the four numbers that started with the same digit, he started by typing that digit. If the computer told him that this digit was incorrect, these two numbers were eliminated at once and only two additional numbers remained to be examined. If that digit was correct, he needed to divide by 9 only the two numbers that started with that digit. To illustrate: given the exercise "423, 610, 498, 536", he examined first the 610 (and eliminated it before typing because "only 63 is divisible by 9") and then he typed a "4" (first digit of both 423 and 498). The computer accepted the 4 and thus he knew that the correct answer was one of these two numbers. He promptly divided 423

by 9 mentally and produced the correct answer within the time limit.

8. Inducing from worked examples, using trial-and-error

When all the previously listed strategies do not work, good students resort to getting help from the computer. They then either press any key three times (for three trials) or they press a particular "advance" key. Both actions result in a display of the correct answer with all intermediate steps. This answer remains on the screen for only a short time (from 6 to 9 seconds). The students concentrate on the displayed answer, trying to identify the algorithm used. Thus, by using the solution displayed by the computer as a worked example, students induce frequently from that example (either from the first one or, using trial-and-error, from several similar examples) the correct algorithm for solution.

Support for this description is provided by students' verbal explanations of their methods for solutions. In their interviews, when asked about how they find the correct solution to an unfamiliar exercise, students' most typical explanations are: "this is the way the computer wants it"; "I tried and it came out right"; "when I solved it in the way I thought I should, I got an error and therefore I had to answer the other way around"; and "the computer showed me how to do this".

Example: A fourth grader who had not yet studied negative numbers received exercises of the type: "Replace the "?" by >, < or = in "-4 ? -19". She explained that when she first received this type of exercises she typed "<" because 4 is less than 19 but "the computer said that she was wrong" and that the correct answer was ">". Since then she knew that every time that she received the same type of exercise, she had to "reverse the logical order"

Through this trial-and-error strategy, it happens that students induce from final answers algorithms that are "local", that is, algorithms that work for that particular type of an exercise but that do not work for the general case.

Example: A bright second grader received a sequence of exercises that required converting mixed numbers into fractions, the integral part of the mixed numbers being the same in all of them--1. For example:

$$1\frac{4}{5} = \frac{?}{5} \qquad 1\frac{2}{3} = \frac{?}{3}$$

The student failed to solve the first two exercises in that sequence but he then solved all the following ones correctly. When the session was over, he explained that for solving 1 4/5 he added 4+5 and wrote 9 for the "?". He did the same with 1 2/3-- replacing the "?" with 5 computed as 2+3 (that is, he did not refer at all to the integral part of the mixed number). This example shows that the boy solved

this sequence of exercises correctly without really understanding the process involved.

9. Acquiring human help

All the highly motivated students that we observed were not ready to accept failure in identifying answers to problems. If they did fail during the computer session, they tried to get the teacher's help, and sometimes to get help from their neighbours (classmates). If they did not succeed in receiving the required algorithm for solution, they saw to it that they remembered the problematic exercises and they looked aggressively for help off-line. They used several sources for off-line help: In-school they asked the teacher during arithmetic lessons or during breaks between the lessons. They also discussed their problems with their classmates, usually during the breaks between lessons. Outside school they asked family members and friends for help.

How do good students acquire their knowledge?

We have seen thus far a variety of strategies that high-achieving students use to advance in the hierarchical levels of CAI practice, very frequently with no understanding of the underlying concepts involved. However, after a while, in their interviews, they demonstrate good understanding of these concepts. How do these students learn new concepts from final answers to exercises without

receiving any organized, orderly, structured instruction? This question should be furthermore investigated. I may only assume that some part of the understanding of the new concepts is developed through the practice itself, and the rest is acquired through obtaining human help. The better students are curious human beings and usually like to understand what they are doing. Thus, when they ask for external help--the teacher, parents, and siblings, these students ask not only for the techniques but also for explanations--for the why's.

Discussion

The designers of the CAI system involved here created a special learning environment, different from the classroom environment, for enhancing individualization of students' work (this goal is stated in the introduction to teachers' handbook in the CBPA curriculum). At face value, students' work with the CBPA system seems to be very dull. The screen display is B&W with almost no graphics and no animation. Thus, it does not include essential features believed to provide motivation for students' learning that are found in the newer microcomputer software D&P programs. However, as we see in this article, the highly individualized curriculum leads to situations and consequences not foreseen by the system's designers. We see here a flourish of resourceful problem-solving strategies by high achievers for mastering skills technically. These skills are learned from material that is designed to provide practice of already known

material rather than to teach new material. However, this study suggests that good students learn not only the techniques for solving the exercises, but eventually they acquire a good understanding of the concepts involved. In contrast, evidence from observations of medium-to-low achieving students (Hativa, 1988a, 1988b) suggests that many of these latter students, when getting material that either has not yet been taught in class or that they have already forgotten, do not acquire full understanding following their mastering of the techniques. However, I believe that even mastering skills without understanding bears positive prospects for facilitating their future learning. It is possible that when the class teacher eventually presents this material in class, it will be easier for these students to understand it because they have already mastered the underlying technique for solution. This point needs further investigation.

This article describes the multiple strategies that high-achieving students use to self-regulate their learning in this CAI-induced learning environment. Self-regulated learning is defined as a student's active acquisition and transformation of instructional material (Mandinach, 1984). A similar phenomenon was observed in other contexts as well (Siegler, 1986). Siegler contends that there are good reasons for students to know and to use a variety of strategies. Strategies differ in their accuracy, in the amounts of time they require, in their memory demands, and

in the range of problems to which they apply. Strategy choices involve trade-offs among these properties so that pupils can cope with cognitive and situational constraints. The broader the range of strategies that children know, and the more effective their procedures for choosing which strategy to use in a given situation, the better they can adapt to the demands of changing circumstances."

In order for students to be able to effectively use a wide range of strategies in practicing with the CBPA system material new to them, students need to have most of the following aptitudes (Hativa, 1988a): quick thinking (the final solutions stay only a short time on the screen); very good induction and deduction abilities; very good memory capacity; high motivation; competitive spirit; persistence; and aggressiveness in looking for answers. Further research should take place to investigate how to teach medium to low achieving students to use strategies and what another type of support these students should receive for effective learning in a CAI environment which is different from class environment.

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